Nonsingular Terminal Sliding Mode Based Adaptive Finite-time Consensus Protocol for Multiple Euler-Lagrange Systems*

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***Abstract*—This paper presents a distributed finite-time consensus protocol for a group of cloud-connected Euler-Lagrange systems with the presence of disturbances. The consensus protocol is designed by using nonsingular terminal sliding-mode-based consensus surface vectors combining position and velocity error vectors. The consensus error states use the position and velocity signals of the local and neighbouring Lagrange systems. The robust adaptive learning algorithm is used to learn and compensate for bounded disturbances associated with parameters and other external disturbance disturbances. Nonsingular terminal sliding mode and Lyapunov theory are used to show the convergence analysis of the proposed protocol. The convergence analysis uses Laplacian matrices and Graph theory to show that the states of a group of nonlinear follower Lagrange systems can reach an agreement by sharing their states to track the states of the leader Lagrange system in finite time. The proof of convergence has three parts. First, analysis shows that the consensus error states are continuous and the parameters are continuous and bounded by compact sets. Second, it shows that the sliding mode motion occurs for each Lagrange system in finite time. Third, convergence analysis shows the finite-time consensus tracking of the position and velocity consensus error states. The design and analysis do not use the exact bound of the disturbances associated with the follower Euler-Lagrange systems. The nonsingular terminal sliding mode-based finite-time consensus protocol can provide faster and more robust convergence against disturbances as opposed to the asymptotic consensus protocol.**

I. Introduction

Over the past years, there has been a great deal of research interest in developing distributed coordination protocols for the formation and tracking of a group of linear and nonlinear systems. Most recent reported results in this area can be traced from the literature, e.g., [1]-[13], [15]-[26], and references therein. A distributed coordination control algorithm has also been applied for highly uncertain Euler-Lagrange systems. The reported designs can be classified into two categories asymptotic/exponential and finite-time convergence-based distributed coordination control algorithms. In asymptotic convergence, the systems can share their state information such that the systems can perform the desired task cooperatively as the time goes to infinity. In finite-time protocol, the systems states can share their information and achieve the desired task cooperatively in finite-time.

In recent years, the consensus-based protocol has also been used to develop distributed control protocols for leader-follower and leaderless multiple Euler-Lagrange systems. Recent results in consensus-based coordination protocol designs for Euler-Lagrange systems can be found in [28]-[31]. The vast majority of the reported consensus-based distributed coordination protocols can ensure asymptotic convergence. However, in some practicable applications, such as disaster response, target surveillance, industrial manipulation, and other cooperative tasks, the consensus among Euler-Lagrange systems may require high performance and more precision coordination control to guarantee faster and finite-time convergence of the closed-loop systems. To ensure a finite-time convergence, the nonsingular terminal sliding mode (NTSM) based finite-time

consensus protocol can be designed for a group of cloud-connected Euler-Lagrange systems. Finite-time consensus protocol allows the systems to share their state's information and achieve the desired task cooperatively in a finite time. In our view, very few NTSM-based finite-time consensus protocols have been reported for Euler-Lagrange systems in the presence of disturbances. Therefore, this work introduces an NTSM-based consensus protocol to solve the finite-time tracking control problem for a group of connected Euler-Lagrangian systems with the presence of disturbances.

In this work, the main goal is to develop nonsingular terminal sliding mode-based finite-time consensus protocols for a group of connected Euler-Lagrange systems against disturbances. The protocol for each Lagrange system is developed by comprising a nonsingular terminal sliding mode surface with robust adaptive learning algorithms. The consensus surface is constructed by combining the position and velocity signals of the local and neighbouring Lagrangian systems. Adaptive learning algorithm introduces learning and adapts with the bounded disturbances associated with Lagrangian systems. The protocol design and convergence analysis are presented with the help of Lyapunov, NTSM and adaptive control theory. The convergence analysis ensures finite-time consensus property on the sliding mode surface provided that the interaction communication topology has a directed spanning tree. The analysis has three parts. The analysis first proves that the consensus error states for a group of cloud-connected leader-follower Lagrangian systems are continuous and bounded provided that the parameters are bounded over the given compact sets. Then, the analysis shows that the sliding mode motion takes place for each Lagrange system in finite time. Finally, the analysis ensures that the position and velocity consensus error states of the leader-follower Lagrangian systems can achieve finite-time consensus tracking properties. The protocol design and analysis do not use the exact bound of the disturbances associated with modelling errors and other external disturbances. The consensus protocol based on NTSM can ensure robustness and faster convergence properties as compared with other consensus-based protocols.

The rest of the paper is organized as follows. Section II provides assumptions, lemmas and definitions related to Laplacian and Graph theory for interaction model among Lagrange systems. Section III presents the dynamical model, property and assumption of the nonlinear Lagrange system. Section III also presents protocol design and convergence analysis. Finally, the paper is concluded in section IV.

II. Definition, Lemmas, Assumptions

The NTSM-based consensus protocol design uses Graph and Laplacian theory to model the information exchange between Lagrangian systems. So, this section provides a brief review of the graph and Laplacian theory and can be traced from [1], and references therein. First, it is assumed that there exist N connected Euler-Lagrangian systems that lie to a finite set $\ell_L = \{1, ..., N\}$. The information exchange topology between N Lagrangian systems can be modelled by a weighted graph as $\mathcal{R} = \{V, E, A\}$ where $V = \{v_i, i \in \ell_L\}$ is the set of nodes. $E \subseteq V \times V$ describes the set of edge and A defines the weighted adjacent matrix of the graph $\mathcal{R}$ with nonnegative elements a_{ij}. The Laplacian matrix can be computed as

$$\mathcal{L} = \mathcal{B}_l - A \tag{1}$$

where $\mathcal{B}_l = diag\{b_{l1}, ..., b_{lN}\}$ with $b_{li} = \sum_{j=1}^{n} a_{ij} = \sum_{j \in \mathcal{O}_i a_{ij}}$ and $\mathcal{O}_i = \{j \mid (v_i, v_j) \in E\}$. The information exchange between leader and follower Lagrange system can be defined by matrices $B = diag(z_1, ..., z_N)$ with $z_i > 0$ [1]. The NTSM-based consensus design and convergence analysis will use the following Lemmas.

Lemma 1[14]: For nonlinear system $\dot{x} = f_l(x)$ with $f_l(0) = 0$, there exists a continuous and differentiable Lyapunov function $V_L(x)$ closed to a neighborhood of the origin such that $V_L(x)$ is positive definite and $\dot{V}_L(x) \leq -g_l V_L^{c_a}(x)$ with $g_l > 0$ and $0 < c_a < 1$. Then, the origin is locally finite-time stable and $V_L((x)(t))$ reaches to zero at finite-time $t_F^* \leq \frac{V_L(0)^{(1-c_a)}}{g_l(1-c_a)}$ and $V_L(x(t)) = 0$ for all $t \geq t_F^*$.

Lemma 2[13]: It is assumed that the graph has a directed spanning tree only when it has at least one node with a directed path to all other nodes.

Lemma 3[13]: When $v_1, ..., v_N \geq 0$ and $0 < b \leq 1$, then

$$\left(\sum_{i=1}^{N} v_i\right)^b \| \leq \sum_{i=1}^{N} v_i^b \tag{2}$$

III. Consensus Protocol Design for Lagrangian System

In this section, we develop an NTSM-based finite-time consensus protocol for leader-follower Euler-Lagrangian systems with the presence of disturbances. We first define a dynamical model for a leader Euler-Lagrangian system by the following equation

$$\mathcal{M}_o(q_o)\ddot{q}_o + \mathcal{C}_o(q_o, \dot{q}_o)\dot{q}_o + \mathcal{T}_o(q_o) \quad = \quad u_o \tag{3}$$

where $\ddot{q}_o \in \Re^n$, $\dot{q}_o \in \Re^n$, $q_o \in \Re^n$ are the acceleration, velocity and position vector, $\mathcal{M}_o(q_o) \in \Re^{n \times n}$ is the symmetric and uniformly positive definite inertia matrices, $\mathcal{C}_o(q_o, \dot{q}_o)\dot{q}_o \in \Re^n$ is the coriolis and centrifugal loading vector, $\mathcal{T}_o(q_o) \in \Re^n$ is the gravitational loading vector, $u_o \in \Re^n$ is the applied control input vector. The dynamical model for a class of Euler-Lagrangian system can be expressed as

$$\mathcal{M}_i(q_i)\ddot{q}_i + \mathcal{C}_i(q_i, \dot{q}_i)\dot{q}_i + \mathcal{T}_i(q_i) \quad = \quad \tau_i + d_i \tag{4}$$

where, $i \in \ell_L$, $\ddot{q}_i \in \Re^n$, $\dot{q}_i \in \Re^n$, $q_i \in \Re^n$ are the acceleration, velocity and position vector, $\mathcal{M}_i(q_i) \in \Re^{n \times n}$ is the symmetric and uniformly positive definite inertia matrices, $\mathcal{C}_i(q_i, \dot{q}_i)\dot{q}_i \in \Re^n$ is the coriolis and centrifugal loading vector, $\mathcal{T}_i(q_i) \in \Re^n$ is the gravitational loading vector, $\tau_i \in \Re^n$ is the applied control input vector. The motion dynamics (4) can also be rewritten with disturbances as

$$\mathcal{M}_{ia}(q_i)\ddot{q}_i + \mathcal{C}_{ia}(q_i, \dot{q}_i)\dot{q}_i + \mathcal{T}_{ia}(q_i) \quad = \quad \tau_i + \mathcal{D}_i \tag{5}$$

with $\mathcal{D}_i = (d_i - \Delta\mathcal{M}_{ia}(q_i) - \Delta\mathcal{C}_{ia}(q_i,\dot{q}_i) - \Delta\mathcal{T}_{ia}(q_i))$. For design and analysis, the motion dynamics (5) are re-written in the following global compact form

$$M_a\ddot{\mathcal{Q}} + C_a\dot{\mathcal{Q}} + G_a \quad = \quad \tau + D \tag{6}$$

where $\mathcal{Q} = [q_1^T, ..., q_N^T]^T$, $D = [\mathcal{D}_1^T, ..., \mathcal{D}_N^T]^T$, $M_a = diag[\mathcal{M}_{1a}, ..., \mathcal{M}_{Na}]$, $C_a = diag[\mathcal{C}_{1a}, ..., \mathcal{C}_{Na}]$, $G_a = diag[\mathcal{T}_{1a}^T, ..., \mathcal{T}_{Na}^T]^T$ and $\tau = diag[\tau_1, ..., \tau_N]^T$.

The design and analysis of the proposed distributed protocol are required to satisfy the following properties and assumptions.

Assumption 1: The input of the leader-Lagrangian system is continuous and bounded as

$$\|u_o\| \leq \beta_o < +\infty \tag{7}$$

Assumption 2: The disturbances associated with the follower Lagrangian systems $\mathcal{D}_i$ is bounded and satisfy the following inequality.

$$\|\mathcal{D}_i\| \leq \alpha_{di} \tag{8}$$

with $\alpha_{di} > 0$.

Assumption 3: The position and velocity of the leader-Lagrangian system is continuous and bounded.

Property 1: The inertial matrices $\mathcal{M}_{ia}$ is a symmetric, positive definite and bounded as $m_{1i} \leq \|\mathcal{M}_{ia}\| \leq m_{2i}$ with $m_{1i} > 0$ and $m_{2i} > 0$.

Property 2: There exists positive constants k_{gi} such that the gravity vectors are bounded as $\|\mathcal{T}_{ia}\| \leq k_{gi}$.

Property 3: The matrices $\mathcal{C}_{ia}$ are bounded as $\|\mathcal{C}_{ia}\| \leq k_{ci}\|\dot{q}_i\|$ with $k_{ci} > 0$.

Property 4: The inertial matrices $\mathcal{M}_o$ is a symmetric, positive definite and bounded as $m_{1o} \leq \|\mathcal{M}_o\| \leq m_{2o}$ with $m_{1o} > 0$ and $m_{2o} > 0$.

Property 5: Since the position and velocity states of the leader-Lagrange system is continuous and bounded, then the matrices $\mathcal{C}_o$ are also bounded as $\|\mathcal{C}_o\| \leq k_{co}\dot{q}_o \leq k_{co}\alpha_{qo}$ with $k_{co} > 0$ and $\alpha_{qo} > 0$.

Property 6: Then gravitational loading vector for the leader Lagrange system is bounded as $\|\mathcal{T}_o\| \leq \alpha_{go}$ with $\alpha_{go} > 0$.

A. Main Results-Design and Convergence Analysis

This subsection develops the distributed protocol for a group of connected Euler-Lagrangian nonlinear systems to achieve finite-time consensus property. For the design, the position and velocity of the leader-Lagrangian system are only available to its neighbouring follower Lagrangian systems. The goal in finite-time consensus is to show that, for any initial conditions, there exists a finite-time t_F^* such that the states of a group of connected follower Euler-Lagrangian systems can share their states and reach an agreement to achieve finite-time tracking as

$$\lim_{t\to t_F^*} \|1_n \otimes q_o(t) - \mathcal{Q}_i(t)\| = 0$$

$$\lim_{t\to t_F^*} \|1_n \otimes \dot{q}_o(t)(t) - \dot{\mathcal{Q}_i}(t)\| = 0$$

Using local neighbourhood position and velocity information, it is possible to design consensus tracking error for the i-th Lagrangian system as

$$\xi_{pi} \quad = \quad \sum_{j=1,j\neq i}^{N} a_{ij}(q_i - q_j) + b_i(q_i - q_o) \tag{9}$$

$$\xi_{vi} \quad = \quad \sum_{j=1,j\neq i}^{N} a_{ij}(\dot{q}_i - \dot{q}_j) + b_i(\dot{q}_i - \dot{q}_o) \tag{10}$$

The consensus tracking error functions for the whole Lagrangian networks can be written in compact column stack vectors as

$$E_1 \quad = \quad ((\mathcal{L} + B) \otimes I_m)\,(1 \otimes q_o - \mathcal{Q}) \tag{11}$$

$$E_2 \quad = \quad ((\mathcal{L} + B) \otimes I_m)\left(1 \otimes \dot{q}_o - \dot{\mathcal{Q}}\right) \tag{12}$$

with $E_1 = [\xi_{p1}, ..., \xi_{pN}]^T$, $E_2 = [\xi_{v1}, ..., \xi_{vN}]^T$, $\mathcal{Q} = [q_1, ..., q_N]^T$ and $\dot{\mathcal{Q}} = [\dot{q}_1, , ..., \dot{q}_N]^T$. The time derivative of (11)-(12) can be written as

$$\dot{E}_1 \quad = \quad E_2 \tag{13}$$

$$\dot{E}_2 \quad = \quad ((\mathcal{L} + B) \otimes I_m)\left(1 \otimes \ddot{q}_o - \ddot{\mathcal{Q}}\right) \tag{14}$$

Now, we define the NTSM-based consensus surface for the i-th leader-follower Lagrangian system as

$$s_i \quad = \quad \xi_{pi} + \alpha_o e_{2i}^{\beta} Sgn(\xi_{vi}) \tag{15}$$

with $i \in \ell_L$, $\alpha_o > 0$ and $1 < \beta < 2$. The NTSM dynamics can be written in the following compact form.

$$S \quad = \quad E_1 + \alpha_o E_2^{\beta} Sgn(E_2) \tag{16}$$

with $S = [s_1, s_2, ..., s_N]^T$. Now, take the derivative (16) and then use (14) to derive $\dot{S}$ as

$$\dot{S} \quad = \quad E_2 + \alpha_o\beta diag(E_2^{\beta-1}t)\Big[((\mathcal{L} + B) \otimes I_m)\left(1 \otimes \ddot{q}_o - \ddot{\mathcal{Q}}\right)\Big] \tag{17}$$

Then, the main result for this paper is presented by the following Theorem 1.

Theorem 1 : Consider a group of cloud-connected leader-follower Euler-Lagrangian systems (17) satisfying assumptions 1-3, properties 1- 6, and lemmas 1 to 3. Then, there exist nonsingular terminal sliding vectors (16) and protocol (22) with estimates (23) such that the parameter estimates are bounded and all the states of the followers Euler-Lagrangian systems can reach an agreement and achieve finite-time consensus property on the NTSM based consensus surface.

Proof: The proof of Theorem 1 can be shown by three parts [26]. The first part proves that the sliding surface is bounded provided that the parameter estimates are continuous and bounded. This also ensures the upper boundedness property of the parameter estimation errors. To prove that, the following Lyapunov function candidate is chosen.

$$V_1 \quad = \quad \frac{S^T S}{2} + \frac{\tilde{\Theta}_1^T \Gamma_1^{-1} \tilde{\Theta}_1}{2} \tag{18}$$

where $\tilde{\Theta}_1 = \Theta_1 - \hat{\Theta}_1$, Γ_1^{-1} is a symmetric, positive-definite and constant diagonal stack matrices Γ_{1i}^{-1}, Θ_1, $\tilde{\Theta}_1$ and $\hat{\Theta}_1$ are the column stack vector of Θ_{1i}, $\tilde{\Theta}_{1i}$ and $\hat{\Theta}_{1i}$ $\forall i \in \ell_s$. Take the derivative (18) along the trajectory of (17). Then, $\dot{V}_1$ can be derived as

$$\dot{V}_1 = S^T\Big[E_2 + \alpha_o\beta diag(E_2^{\beta-1})\Big(((\mathcal{L}+B)\otimes I_m)(1\otimes\ddot{q}_o - \ddot{\mathcal{Q}})\Big)\Big] + \tilde{\Theta}_1^T\Gamma_1^{-1}\dot{\tilde{\Theta}}_1 \quad (19)$$

Using properties 4-6 and assumptions 1 & 3, the term $1\otimes\ddot{q}_o$ in (19) can be derived as

$$\Big\|1\otimes\mathcal{M}_o^{-1}(-\mathcal{C}_o\dot{q}_o - \mathcal{T}_o + u_o)\Big\| \leq \Big[\Big(\alpha_{mo}k_{co} \alpha_{qo} + \alpha_{mo}\alpha_{go} + \alpha_{mo}\beta_o\Big)\Big] \leq \gamma_l \quad (20)$$

where $\|\mathcal{M}_o^{-1}\| = \alpha_{mo}$, $\gamma_l = \alpha_{mo}\alpha_l$ and $\alpha_l = (k_{co}\alpha_{qo} + \alpha_{go} + \beta_o)$. In view of properties 1-3, and assumption 2, the term $\ddot{\mathcal{Q}}$ in (19) can be expressed as

$$\Big\|M_a^{-1}(C_a\dot{\mathcal{Q}} + G_a - \tau - D)\Big\| \leq \alpha_m\Big(k_c a_1\|\dot{\mathcal{Q}}\| + k_c a_2 k_g + \alpha_d - \tau\Big) \leq \Big(\alpha_m k_c a_1\|\dot{\mathcal{Q}}\| + \alpha_m k_c a_2 + \alpha_m k_g + \alpha_m\alpha_d - \alpha_m\tau\Big) \leq \Big(\delta_1\|\dot{\mathcal{Q}}\| + \delta_2 - \alpha_m\tau\Big) \quad (21)$$

with $\|M_a^{-1}\| = \alpha_m$, $\alpha_m > 0$, $\|\dot{\mathcal{Q}}\|^2 \leq a_1\dot{\mathcal{Q}} + a_2$, $a_1 > 0$, $a_2 > 0$, $\delta_1 = \alpha_m k_c a_1$, $\delta_2 = (\alpha_m k_c a_2 + \alpha_m k_g + \alpha_m\alpha_d)$. Then, using (20)-(21) together with Kronecker product, $\dot{V}_1$ can be written as

$$\dot{V}_1 \leq S^T\Big[E_2 + \alpha_o\beta diag(E_2^{\beta-1})\Big(\|(\mathcal{L}+B)\|(\mathcal{Y}_l\Theta_1 - \alpha_m\tau)\Big)\Big] + \tilde{\Theta}_1^T\Gamma_1\dot{\tilde{\Theta}}_1 \quad (22)$$

where $\Theta_1 = \gamma_L,\ \delta_2]^T$ with $\gamma_L = (\gamma_l + \delta_2)$ and $\mathcal{Y}_l = [\|\dot{\mathcal{Q}}\|\ sign(S)]$. Now, using the following protocol together with adaptive learning laws

$$\tau = \alpha_m^{-1}\Big((\alpha_o\beta)^{-1}\|(\mathcal{L}+B)\|^{-1}\|diag(E_2^{(2-\beta)})\| + \mathcal{Y}_l\hat{\Theta}_1 + k_L\|S\|\Big) \quad (23)$$

$$\dot{\tilde{\Theta}}_1 = -\Gamma_1\alpha_o\beta\mathcal{Y}_l^T\|(\mathcal{L}+B)\|\|diag(E_2^{(\beta-1)})\|\|S\| \quad (24)$$

$\dot{V}_1$ can be written as

$$\dot{V}_1 \leq -\eta_y\|S\|^2 \leq 0 \quad (25)$$

where $\eta_y = min\Big\{k_L\alpha_o\beta\|diag(E_2^{(2-\beta)})\|\|(\mathcal{L}+B)\|\Big\}$ with $\|E_2\| \neq 0$. Now, using (25), one can conclude from the Lyapunov theorem that all the signals of the leader-followers Lagrangian closed loop systems are bounded. The parameter estimates are also bounded provided that there exist positive constants Θ_1^* such that $\hat{\Theta}_1 \leq \Theta_1^*$. This ensures the upper bounded property stated in the first part.

The second part proves that the sliding motion is achieved as $S = 0$ within a finite time. To do that, the following Lyapunov function candidate is selected.

$$V_2 = \frac{1}{2}S^TS + \frac{\tilde{\Theta}_1^T\Gamma_{1o}\tilde{\Theta}_1}{2} \quad (26)$$

where Γ_{1o} is a symmetric, positive definite, and constant diagonal stack matrices Γ_{1oi}^{-1}. Now, using protocol (23)-(24), $\dot{V}_2$ can be written as

$$\dot{V}_2 \leq -\eta_y\|S\|^2 + \|S\|\alpha_o\beta\|diag\left(E_2^{\beta-1}\right)\|\|(\mathcal{L}+B)\| \mathcal{Y}_l\Theta_1 - S^T\alpha_o\beta\|diag\left(E_2^{\beta-1}\right)\|\|(\mathcal{L}+B)\|\mathcal{Y}_l\hat{\Theta}_1 - \tilde{\Theta}_1^T\Gamma_{1o}\Gamma_1\alpha_o\beta\mathcal{Y}_l^T\|(\mathcal{L}+B)\|\|\|diag(E_2^{(\beta-1)})\|\|S\| \quad (27)$$

Equation (27) can be re-written as

$$\dot{V}_2 \leq -\eta_y\|S\|^2 - \|S\|\alpha_o\beta\|diag\left(E_2^{\beta-1}\right)\|\|(\mathcal{L}+B)\| \mathcal{Y}_l(\Theta_1^* - \Theta_1) + \|S\|\alpha_o\beta diag\left(E_2^{\beta-1}\right)\|(\mathcal{L}+B)\| \mathcal{Y}_l\left(\Theta_1^* - \hat{\Theta}_1\right) - \left(\Theta_1^* - \hat{\Theta}_1\right)\Gamma_{1o}\Gamma_1\alpha_o\beta\mathcal{Y}_l^T \|(\mathcal{L}+B)\|\|\|diag(E_2^{(\beta-1)})\|\|S\| \quad (28)$$

Now, using $\xi_F = \alpha_o\|(\mathcal{L}+B)\|\beta\|diag\left(E_2^{\beta-1}\right)\|$ and $\mathcal{A}_F = \Gamma_{1o}\Gamma_1\alpha_o\|(\mathcal{L}+B)\|\beta\mathcal{Y}_l^T\|diag(E_2^{(\beta-1)})\|\|S\|$, $\dot{V}_2$ has the following form.

$$\dot{V}_2 \leq -\xi_F\mathcal{Y}_l(\Theta_1^* - \Theta_1)\|S\| - (\mathcal{A}_F - S\xi_F\mathcal{Y}_l)\left(\Theta_1^* - \hat{\Theta}_1\right) \quad (29)$$

Equation (29) can also be written as

$$\dot{V}_2 \leq -\sqrt{2}\xi_F\mathcal{Y}_l(\Theta_1^* - \Theta_1)\frac{\|S\|}{\sqrt{2}} - \sqrt{2\Gamma_{1o}}\Big((\mathcal{A}_F - S\xi_F\mathcal{Y}_l)\Big)\frac{\left(\|\Theta_1^* - \hat{\Theta}_1\|\right)}{\sqrt{2\Gamma_{1o}}} \quad (30)$$

Define $\mathcal{A} = \xi_F\mathcal{Y}_l(\Theta_1^* - \Theta_1)$ and $\mathcal{B} = \sqrt{\Gamma_{1o}}(\mathcal{A}_F - S\xi_F\mathcal{Y}_l)$. Then, $\dot{V}_2$ can be simplified as

$$\dot{V}_2 \leq -\sqrt{2}\mathcal{A}\frac{\|S\|}{\sqrt{2}} - \sqrt{2}\mathcal{B}\frac{\left(\|\Theta_1^* - \hat{\Theta}_1\|\right)}{\sqrt{2\Gamma_{1o}}} \quad (31)$$

Then, using $\mathcal{Z} = min\{\mathcal{A}, \mathcal{B}\}$, one can write $\dot{V}_2$ in the following simplified form.

$$\dot{V}_2 \leq -\sqrt{2}\mathcal{Z}\left(\frac{\|S\|}{\sqrt{2}} + \frac{\left(\|\Theta_1^* - \hat{\Theta}_1\|\right)}{\sqrt{2\Gamma_{1o}}}\right) \quad (32)$$

Now, by choosing $\Theta_1^* > \Theta_1$, $\|\Gamma_{1o}\| < \|\Gamma_1^{-1}\|$, one has $\mathcal{A} > 0$, $\mathcal{B} > 0$, $\mathcal{Z} > 0$. Then, applying Lemma 3 and (26), $\dot{V}_2$ can be simplified further

$$\dot{V}_2 \leq \sqrt{2}\mathcal{Z}V_2^{\frac{1}{2}} \quad (33)$$

For $E_2 \neq 0$, the bound on $\|diag\left(E_2^{\beta-1}\right)\| > 0$. In view of Lemma 1, Lemma 2, and $\mathcal{Z} > 0$, one can state that the states of the closed systems for a group of connected Lagrange systems under the proposed consensus protocol can reach the sliding surfaces $S = 0$ in finite-time relying on the initial states $S(0)$ with the presence of bounded disturbances. The finite-time can be manipulated $\frac{\sqrt{2}V_2(0)^{\frac{1}{2}}}{\mathcal{Z}}$.

In part 3, we show that the states of the follower Lagrange systems can achieve consensus tracking in finite time on the sliding mode surface with bounded disturbances and remain there for all time. To show that, the following Lyapunov function candidate is chosen.

$$V_3 = \frac{E_1^T E_1}{2} \tag{34}$$

Now, on the sliding surface $S = 0$, one can write

$$E_2 = -\alpha_o^{\beta} E_1^{\frac{1}{\beta}} \tag{35}$$

Then, the time derivative of V_3 can be written as

$$\dot{V}_3 = -\alpha_o^{\beta} E_1^T E_1^{\frac{1}{\beta}} \tag{36}$$

Now, applying

$$E_1^T E_1^{\frac{1}{\beta}} \quad = \quad \sum_{i=1}^{N}\sum_{j=1}^{m} \left(\xi_{p1ij}\right)^{2\frac{(\beta+1)}{2\beta}} \tag{37}$$

$\dot{V}_3$ can be written in the following compact form

$$\dot{V}_3 \quad \leq \quad -\alpha_o^{\beta} 2^{\frac{(\beta+1)}{2\beta}} V_3^{\frac{(\beta+1)}{2\beta}} \tag{38}$$

Then, using $\beta < 2$, it can be stated that there exists a finite time depending on the initial states with $t_o > \frac{\sqrt{2}V_2(0)^{\frac{1}{2}}}{\mathcal{Z}}$ such that the errors E_1 will converge to zero in finite time.

Since E_1 converges to zero, then, using Lemma 2, one can also state that E_2 converges to zero in finite time. This means that the velocity states $\dot{\mathcal{Q}}$ for a group of Lagrange systems can converge to the velocity states of the leader Lagrange system $1 \otimes \dot{q}_o$ in finite-time. Then, using Lyapunov theory, one can conclude that the position states for a group of connected Lagrange systems $\mathcal{Q}$ can track the position states of the leader Lagrange system $1 \otimes q_o$ in finite-time provided that the adaptive learning laws for parameters are bounded for the given compact sets.

Remark 1: The estimates $\hat{\Theta}_1$ (23) may exhibit discontinuity. To ensure smooth parameter estimates, the estimates $\hat{\Theta}_1$ (24) can be adjusted with the smooth parameter projection scheme $\dot{\hat{\Theta}}_k = [Proj(\hat{\Theta}_1, \Phi)]_k$ [27].

Remark 2: The design uses $sgn(.)$ function which may generate a chattering phenomenon. To attenuate the chattering phenomenon, one can estimate the $sgn(.)$ by using $sat(.)$ function. The $sat(.)$ is considered to be a bounded saturation function that satisfies

$$sat\left(s_i\right) = \begin{cases} -1 & s_i < -b \\ \frac{s_i}{b} & |s_i| \leq b \\ 1 & s_i > b \end{cases}$$

By using $sat(.)$, it is also possible to guarantee the finite-time convergence as derived in Theorem 1 [26].

IV. CONCLUSION

In this paper, an NTSM-based finite-time consensus tracking protocol has been presented for a group of connected Euler-Lagrangian nonlinear systems. The protocol design used nonsingular terminal sliding mode-based consensus surfaces that are comprised of the position and velocity signals of the local and neighbouring Lagrangian systems with directed interaction topology. The protocol design and convergence analysis have been given by using Lyapunov and nonsingular terminal sliding mode control theory against disturbances associated with the unmodeled dynamics and other external disturbances. The analysis has three parts. The first part of the analysis proved that the error states of the closed-loop leader-follower Euler-Lagrange systems with the presence of disturbances are bounded by constant over the given compact sets. The second part showed that the sliding mode motion occurred for each Lagrangian system in finite time. The third part ensured finite-time consensus convergence of the position and velocity error states provided that the interaction topology has a directed spanning tree. The design and convergence analysis does not demand the exact bound of the disturbances associated with the modelling errors and other external disturbances. In contrast with the asymptotic consensus protocol, the NTSM-based finite-time consensus protocol can ensure faster and more robust state convergence with the presence of bounded disturbances.

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